دراسة مقارنه بين التقديرات البيزيه وتقديرات الإمكان الأكبر لمعالم توزيع بيتا ـ معكوس وايبل

Comparison Study Between the Maximum Likelihood and Bayesian Estimation for the Beta Inverse Wiebull Distribution

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تم اشتقاق توزيع معكوس وايبل ١٩٨٢ علي يد كيللر وكاماث عند وجود منحني للأعطال في محركات الديزل يختلف عن منحنيات توزيعات الصلاحية المعروفة سابقا مثل التوزيع الآسي وتوزيع وايبل.

ومنذ اكتشافه تم اعتباره من أهم التوزيعات الاحتمالية وأكثرها تطبيقا في دراسة صلاحية الأجزاء الاستاتيكيه من الأجهزة وكثر استخدامه لدراسة الصلاحية والفشل في العديد من العلوم والتطبيقات الحيوية مثل الهندسة والطب.

وفي محاوله لزيادة استخدامه وإعطائه مرونة أكثر اشتق خان ٢٠١٠ توزيع بيتا – معكوس وايبل عن طريق لوغاريتم معكوس بيتا وايبل حتى يمكن استخدامه في الأجزاء الميكانيكية والاستاتيكية معا.

يهدف البحث إلى دراسة معالم التوزيع (معلمة الوزن ومعلمة الشكل) وإيجاد تقديرات لها باستخدام أسلوب الإمكان الأكبر كأسلوب كلاسيكي وطريقة بييز كأسلوب حديث وعمل مقارنه بينهم لاختيار أفضل أسلوب تقدير فيما بينهم

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- تم إيجاد تقديرات لمعالم توزيع بيتا – معكوس وايبل باستخدام طريقه بييز (تحت عدد من دوال الخسارة المتماثلة والغير متماثلة).
- تم إيجاد مقدرات الإمكان الأكبر لمعالم التوزيع
- تم عمل مقارنه رقمية بينهم باستخدام البرنامج الإحصائي MathCAD واختيار أفضل أسلوب عن طريق اقل متوسط لمربعات الخطأ .
- تم رسم منحنيات التوزيع عند قيم مختلفة للمعالم

ABSTRACT

The inverse Weibull distribution which was presented by Keller and Kamath (١٩٨٢), this distribution was derived on the basis of physical considerations of failure of dynamic mechanical components subject to degradation phenomena. The system fails especially when the load pressure exceeds the stress resisting capacity (strength) of the component. A generalization of inverse Weibull distribution referred to as the Beta Inverse Weibull distribution (BIW) which is generated from the logit of beta random variable was introduced by Khan (٢٠١٠) as a life time distribution to give more flexibility to the inverse Weibull distribution. The Bayesian estimators (under the complete linear exponential (LINEX) and general entropy loss functions) and maximum likelihood estimator to the unknown parameters $\alpha, \beta, a and b$ will be introduced.

Keywords: Beta Inverse Weibull distribution, Bayesian Estimation, maximum likelihood estimation , (LINEX) loss function, inverted gamma distribution.

١. *INTRODUCTION*

As noted by Keller and Kamath (١٩٨٢) and Calabria and Pulcini (١٩٨٩,٩٠), the maximum likelihood estimates to the scale and shape parameters of the inverse Weibull distribution in the case of complete sampling can be obtained by using the same two equations derived for the parameters of the Weibull distribution. Thus the maximum likelihood estimator's scale and shape have the same statistical properties of the corresponding estimators of the Weibull distribution.

Erto (١٩٨٩) showed that the Inverse Weibull distribution provides a good fit to several data given in literature, such as times to breakdown of an insulating fluid subject to the action of a constant tension. Also, Drapella (١٩٩٣) and Jiang et al. (٢٠٠١) introduced graphical plotting techniques, Yahgmae et al . (٢٠١٣) presented a Beyesian estimation to the scale parameter of inverse Weibull distributionusing quasi, gamma, and uniform priors distributions under the square error, entropy, and precautionary loss functions.

In this paper, the focus of our attention is concentrated on the generalization of the inverse Weibull distribution referred to as the Beta inverse Weibull distribution which is generated from the logit of a beta random variable. Generalized beta distributions have been widely studied in statistics and numerous authors have developed various classes of these distributions. One major benefit of the class of beta generalized distributions is its ability of fitting of skewed data that cannot be properly fitted by existing distributions.

We will obtain the estimators of the unknown parameters for BIW distribution using maximum likelihood and Bayesian estimation methods. The aim of likelihood estimation is to determine the estimates for the parameters, to define a sequence of roots of the likelihood equation that is consistent and asymptotically of The method of maximum likelihood (MLE) estimation is applicable mainly in situations where the true distribution is known apart from the values of a finite number of unknown real parameters efficient.

Generalized class of probability distributions discussed by Eugene et al. (٢٠٠٢). Let $G(y)$be the cumulative distribution function (cdf) of a random variable Y . The cdf's for a generalized class of distributions for the random variable Y , defined by Eugene et al. (٢٠٠٢) as the logit of beta random variable, is given as:

$$F(y) = I_{G(y)}(a,b) a > 0 \quad and b > 0. \qquad (١)$$

Where

$$I_{G(y)}(a,b) = \frac{B_{G(y)}(a,b)}{B(a,b)} \quad \text{and} \quad B_{G(y)}(a,b) = \int_0^{G(y)} t^{a-1}(1-t)^{b-1}\,dt. \qquad (٢)$$

The BIW distribution was first introduced by Khan (٢٠١٠) as a new reliability model by taking $G(y)$ to be the cdf of the inverse Weibull distribution.

Eugene et al. (٢٠٠٢) introduced the Beta normal distribution by taking $G(y)$ to be the cdf of the normal distribution. The only properties of the beta normal distribution known are some first moments derived by Eugene et al. (٢٠٠٢). Cordeiro et al. (٢٠٠٨) proposed the Beta generalized exponential (BGE) distribution which generalizes the beta exponential distribution discussed by Nadarajah and Kotz (٢٠٠٥) and the generalized exponential (also named exponentiated exponential) distribution introduced by Gupta and Kundu (١٩٩٩). They provided a comprehensive mathematical treatment of BGE distribution with the hope that this generalization might attract wider applications in reliability and biology.

Kersey (٢٠١٠) analyzed the behavior of the probability density function by plotting the probability density function for some fixed values of the parameters and he derived the moments and moment generating function for this distribution. He also obtained very useful transformations which showed him the relationships between beta distribution and inverse beta distribution, Beta distribution and BIW distribution that provide a way to generate data from the BIW distribution.

٢. THE BETA INVERSE WEIBULL PROBABILITY DENSITY FUNCTION

The probability density function (pdf) of the Beta inverse Weibull (BIW) distribution is given by(Kersey ٢٠١٠) as follows:

$$f(y) = \frac{\beta\alpha^{-\beta}}{\beta(a,b)} y^{-(\beta+1)} e^{-a(\alpha y)^{-\beta}} \left[1 - e^{-(\alpha y)^{-\beta}}\right]^{b-1}, \text{ for}$$

$y \geq 0\ , \alpha > 0, \beta > 0, a > 0$ and $b > 0.$ (٣)

The pdf of the BIW distribution in (٣) also can be found by using this transformation $Y = \frac{[-log_e(x)]^{-\frac{1}{\beta}}}{\alpha}$ where X is a random variable that follows a beta distribution with parameters *a* and *b* as follows:

$$Y = \frac{[-log_e(x)]^{-\frac{1}{\beta}}}{\alpha}$$

$$g_Y(y) = \frac{\beta\alpha^{-\beta}}{\beta(a,b)} y^{-(\beta+1)} e^{-a(\alpha y)^{-\beta}} \left[1 - e^{-(\alpha y)^{-\beta}}\right]^{b-1}, for\, y \geq 0\ , \alpha > 0, \beta > 0,$$

$a > 0,$ and $b > 0.$

Figures show the differentshapes of the pdf for selected values of the parameters of the BIW distribution.

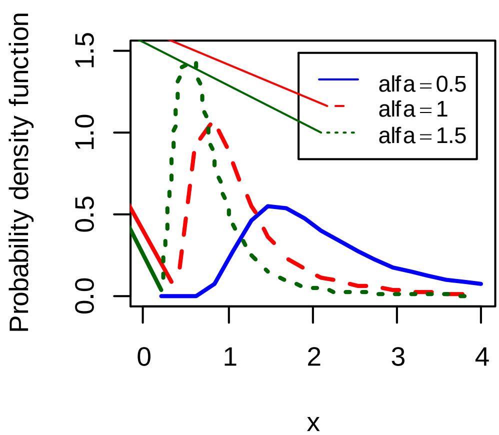

Figure ١.plot of pdf of the BIW distribution with fixed values β, a, b

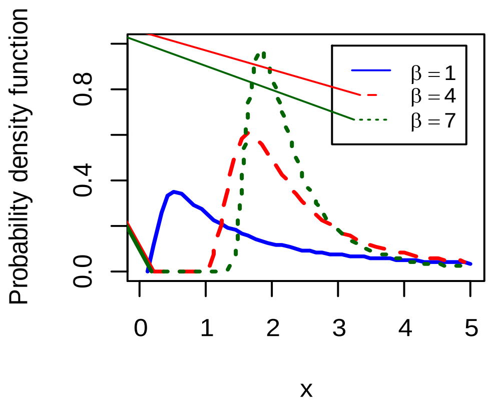

Figure ٢.plot of pdf of the BIW distribution with fixed values α, a, b

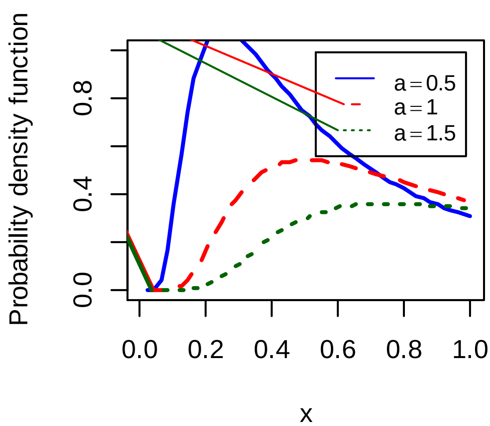

Figure ٣.plot of pdf of the BIW distribution with fixed values α, β, b

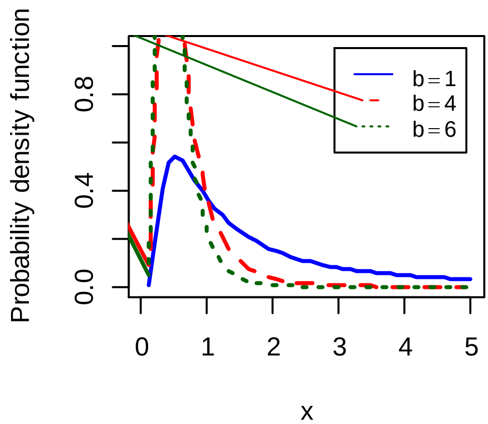

Figure ٤.plot of pdf of the BIW distribution with fixed values a, α, β

٣. Bayesian Estimation of the Parameters

In this section, the Bayesian estimators of unknown parameters function in case of the complete sample for BIW distribution, using non-informative prior distribution based on squared error, linear exponential (LINEX) and general entropy loss functions will be obtained.

The likelihood function has had the following form:

$$L(\underline{x}, \alpha, \beta, a, b) = \left[\frac{1}{\beta(a,b)}\right]^n \frac{\beta^n}{\alpha^{n\beta}} \prod_{i=1}^{n}\left[x_i^{-(\beta+1)}\right]\left(e^{-\sum_{i=1}^{n}(\alpha x_i)^{-\beta}}\right)^a * \prod_{i=1}^{n}\left[1 - e^{-(\alpha x_i)^{-\beta}}\right]^{b-1} ,$$

From (٤) and assumed that the unknown four parametersα, β, a and b have independent non-informative prior distributions defined ,respectively, as follows:

$$\pi_1(\alpha) = \frac{1}{\alpha} \alpha > 0, \qquad (٥)$$

$$\pi_2(\beta) = \frac{1}{\beta}\, \beta > 0, \qquad (٦)$$

$$\pi_3(a) = \frac{1}{a}\, a > 0, \qquad (٧)$$

$$\pi_4(b) = \frac{1}{b}\, b > 0, \qquad (٨)$$

Then, the joint non-informative prior distribution will be given as follows:

$$\pi(\alpha, \beta, a, b) = \frac{1}{\alpha\beta a b}\, \alpha, \beta, a, b > 0. \qquad (٩)$$

The joint posterior distribution of α, β, a and b in case of the complete sample will be defined as follows:

$$\pi(\alpha, \beta, a, b|\underline{x}) = \frac{\pi(\alpha,\beta,a,b) L(\underline{x}|\alpha,\beta,a,b)}{\int\int\int\int_0^\infty \pi(\alpha,\beta,a,b) L(\underline{x}|\alpha,\beta,a,b)\, d\alpha\, d\beta\, da\, db} \qquad (١٠)$$

$$\pi(\alpha, \beta, a, b|\underline{x}) = \frac{A\beta^{n-1}(ab)^{-1}\alpha^{-(n\beta+1)}\, u_i\left(e^{-\sum_{i=1}^{n} z_i}\right)^a \prod_{i=1}^{n}[1-e^{-z_i}]^{b-1}}{M} \qquad (١١),$$

where $A = \left[\frac{1}{\beta(a,b)}\right]^n$, $u_i = \prod_{i=1}^{n}\left[x_i^{-(\beta+1)}\right]$, $z_i = (\alpha x_i)^{-\beta}$ and

$$M = \int\int\int\int_0^\infty \frac{A}{ab}\frac{\beta^{n-1}}{\alpha^{(n\beta+1)}}\, u_i\left(e^{-\sum_{i=1}^{n} z_i}\right)^a \prod_{i=1}^{n}[1 - e^{-z_i}]^{b-1}\, d\alpha\, d\beta\, da\, db\,. \qquad (١٢)$$

The marginal posterior distribution of one parameter will obtained by integrating the joint posterior distribution with respect to the other parameters as follows:

$$\pi_1(\alpha|\beta, a, b, \underline{x}) = \iiint_0^\infty \pi(\alpha, \beta, a, b|\underline{x})\, d\beta\, da\, db \qquad (١٣)$$

$$\pi_2(\beta|\alpha, a, b, \underline{x}) = \iiint_0^\infty \pi(\alpha, \beta, a, b|\underline{x})\, d\alpha\, da\, db \qquad (١٤)$$

$$\pi_3(a|\alpha,\beta,b,\underline{x}) = \iiint_0^\infty \pi(\alpha,\beta,a,b|\underline{x})\, d\alpha d\beta db \qquad (١٥)$$

$$\pi_4(b|\alpha,\beta,a,\underline{x}) = \iiint_0^\infty \pi(\alpha,\beta,a,b|\underline{x})\, d\alpha d\beta da \qquad (١٦)$$

Consequently ,the marginal posterior distribution of α will be

$$\pi_1(\alpha|\beta,a,b,\underline{x}) = \frac{1}{M}\alpha^{-1}\iiint_0^\infty \mathrm{A}\beta^{n-1}(ab)^{-1}\alpha^{-(n\beta)}\, u_i\left(e^{-\sum_{i=1}^n z_i}\right)^a * \prod_{i=1}^n[1-e^{-z_i}]^{b-1}\, d\beta da db \qquad (١٧)$$

Similarly, the marginal posterior distributions of β, a and b,respectively, will be

$$\pi_2(\beta|\alpha,a,b,\underline{x}) = \frac{1}{M}\beta^{n-1}u_i\iiint_0^\infty \mathrm{A}(ab)^{-1}\alpha^{-(n\beta+1)}\, e^{-az_i} * \prod_{i=1}^n[1-e^{-z_i}]^{b-1}\, d\alpha da db \qquad (١٨),$$

$$\pi_3(a|\alpha,\beta,b,\underline{x}) = \frac{1}{M}a^{-1}\iiint_0^\infty \mathrm{A}\beta^{n-1}(b)^{-1}\alpha^{-(n\beta+1)}\, u_i\left(e^{-\sum_{i=1}^n z_i}\right)^a * \prod_{i=1}^n[1-e^{-z_i}]^{b-1}\, d\alpha d\beta db \qquad (١٩),$$

$$\pi_4(b|\alpha,\beta,a,\underline{x}) = \frac{1}{M}b^{-1}\iiint_0^\infty \mathrm{A}\beta^{n-1}(a)^{-1}\alpha^{-(n\beta+1)}\, u_i\left(e^{-\sum_{i=1}^n z_i}\right)^a * \prod_{i=1}^n[1-e^{-z_i}]^{b-1}\, d\alpha d\beta da \qquad (٢٠)$$

١.Bayesian Estimation Under Squared Error Loss Function

Based on squared error loss function , Bayesian estimators $\hat{\alpha}_s, \hat{\beta}_s, \hat{a}_s$ and $\hat{b}_s$ of α, β, a and b, respectively , are given as follows:

$$\hat{\alpha}_s = \frac{1}{M}\iiint_0^\infty \frac{\mathrm{A}\beta^{n-1}}{ab}\alpha^{-(n\beta)}\, u_i\left(e^{-\sum_{i=1}^n z_i}\right)^a \prod_{i=1}^n[1-e^{-z_i}]^{b-1}\, d\beta da db \qquad (٢١),$$

$$\hat{\beta}_s = \frac{1}{M}\beta^n u_i \iiint_0^\infty \frac{A}{ab}\alpha^{-(n\beta+1)} e^{-az_i} \prod_{i=1}^n [1-e^{-z_i}]^{b-1}\, d\alpha da db \qquad (٢٢),$$

$$\hat{\alpha}_s = \frac{1}{M}\iiint_0^\infty \frac{A\beta^{n-1}}{b}\alpha^{-(n\beta+1)} u_i\left(e^{-\sum_{i=1}^n z_i}\right)^a \prod_{i=1}^n [1-e^{-z_i}]^{b-1}\, d\alpha d\beta db \quad (٢٣),$$

$$\hat{b}_s = \frac{1}{M}\iiint_0^\infty \frac{A\beta^{n-1}}{a}\alpha^{-(n\beta+1)} u_i\left(e^{-\sum_{i=1}^n z_i}\right)^a \prod_{i=1}^n [1-e^{-z_i}]^{b-1}\, d\alpha d\beta da \quad (٢٤)$$

The equations (٢١) to (٢٤) can be solved numerically.

١.Bayesian Estimation Under Linear Exponential Loss Function

Based on LINEX loss function , Bayesian estimators $\hat{\alpha}_L, \hat{\beta}_L, \hat{a}_L, \hat{b}_L$ and of α, β, a, b, respectively , are given as follows:

$$\hat{\alpha}_L = -\frac{1}{c}\ln\left[\int_0^\infty e^{-c\alpha}\left[\pi_1(\alpha|\beta,a,b,\underline{x})\right] d\alpha\right] \qquad (٢٥)$$

$$\hat{\beta}_L = -\frac{1}{c}\ln\left[\int_0^\infty e^{-c\beta}\left[\pi_2(\beta|\alpha,a,b,\underline{x})\right] d\beta\right] \qquad (٢٦)$$

$$\hat{a}_L = -\frac{1}{c}\ln\left[\int_0^\infty e^{-ca}\left[\pi_3(a|\alpha,\beta,b,\underline{x})\right] da\right] \qquad (٢٧)$$

and

$$\hat{b}_L = -\frac{1}{c}\ln\left[\int_0^\infty e^{-cb}\left[\pi_4(b|\alpha,\beta,a,\underline{x})\right] db\right] \qquad (٢٨)$$

The equations from (٢٥) to (٢٨) can be solved numerically.

٢.Bayesian Estimation Under General Entropy Loss Function

Based on general entropy loss function , Bayesian estimators $\hat{\alpha}_g, \hat{\beta}_g, \hat{a}_g$ and $\hat{b}_g$ of α, β, a and b, respectively , are given as follows:

$$\hat{\alpha}_g = \left[\int_0^\infty \alpha^{-q}\pi_1(\alpha|\beta,a,b,\underline{x})\, d\alpha\right]^{-\frac{1}{q}} \qquad (٢٩)$$

$$\hat{\beta}_g = \left[\int_0^\infty \beta^{-q}\pi_2(\beta|\alpha,a,b,\underline{x})\, d\beta\right]^{-\frac{1}{q}} \qquad (٣٠)$$

$$\hat{a}_g = \left[\int_0^\infty a^{-q}\pi_3\left(a \middle| \alpha,\beta,b,\underline{x}\right) da\right]^{-\frac{1}{q}} \quad (٣١)$$

and

$$\hat{b}_g = \left[\int_0^\infty b^{-q}\pi_4\left(b \middle| \alpha,\beta,a,\underline{x}\right) db\right]^{-\frac{1}{q}} \quad (٣٢)$$

The equations from (٢٩) to (٣٢) will be solved numerically.

٤.Maximum Likelihood Estimation of the Parameters

In this section, we will obtain the estimators of the unknown parameters in case of the complete sample for BIW distribution, using the maximum likelihood estimation method .

The likelihood function has had the following form:

$$L\left(\underline{x},\alpha,\beta,a,b\right) = \left[\frac{1}{\beta(a,b)}\right]^n \frac{\beta^n}{\alpha^{n\beta}} \prod_{i=1}^{n}\left[x_i^{-(\beta+1)}\right]\left(e^{-\sum_{i=1}^{n}(\alpha x_i)^{-\beta}}\right)^a * \prod_{i=1}^{n}\left[1-e^{-(\alpha x_i)^{-\beta}}\right]^{b-1} \quad (٣٣),$$

to obtain the MLE's of the α,β,a and b we will find the natural logarithm of the likelihood function in (٣٣) as follows:

$$\ln L = n\ln\left[\frac{1}{\beta(a,b)}\right] + n\ln\beta - n\beta\ln\alpha + \sum_{i=1}^{n}\ln x_i^{-(\beta+1)} - a\sum_{i=1}^{n}(\alpha x_i)^{-\beta} + (b-1)\sum_{i=1}^{n}\ln\left[1-e^{-(\alpha x_i)^{-\beta}}\right] \quad (٣٤)$$

By taking the first derivative of $\ln L$ in (٣٤) with respect to α,β,a and $b$and equating to zero, then,

$$\frac{\partial \ln L}{\partial \alpha} = -\frac{n\beta}{\alpha} + \frac{a\beta}{\alpha^{-(\beta+1)}} \sum_{i=1}^{n} x_i^{-\beta} - \frac{\beta}{\alpha^{-(\beta+1)}} (b-1) \sum_{i=1}^{n} \frac{x_i^{-\beta} e^{-(\alpha x_i)^{-\beta}}}{\left[1-e^{-(\alpha x_i)^{-\beta}}\right]} \quad (٣٥)$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} - n \ln \alpha - \sum_{i=1}^{n} x_i + a \sum_{i=1}^{n} (\alpha x_i)^{-\beta} \ln(\alpha x_i) - (b-1) *$$

$$\sum_{i=1}^{n} \frac{(\alpha x_i)^{-\beta} \ln(\alpha x_i) e^{-(\alpha x_i)^{-\beta}}}{\left[1-e^{-(\alpha x_i)^{-\beta}}\right]} \quad (٣٦)$$

$$\frac{\partial \ln L}{\partial a} = -n[\psi(a) - \psi(a+b)] - \sum_{i=1}^{n} (\alpha x_i)^{-\beta} \quad (٣٧)$$

$$\frac{\partial \ln L}{\partial b} = -n[\psi(b) - \psi(a+b)] + \sum_{i=1}^{n} \ln\left[1 - e^{-(\alpha x_i)^{-\beta}}\right] \quad (٣٨)$$

where $\psi(x)$ is the digamma function defined as ,

$$\psi(x) = \frac{d}{dx}[\ln \Gamma(x)] = \frac{\Gamma'(x)}{\Gamma(x)},$$

consequently, the MLE's $\bar{\alpha}, \bar{\beta}, \bar{a}$and $\bar{b}$of parameters for BIW are the solutions of the equations (٣٥-٣٨).

The MLE of reliability function ,$R\left(x \middle| \bar{\alpha}, \bar{\beta}, \bar{a}, \bar{b}\right)$, can be obtained by:

$$\bar{R}\left(t \middle| \bar{\alpha}, \bar{\beta}, \bar{a}, \bar{b}\right) = 1 - \frac{\Gamma(\bar{b})}{\beta(\bar{a}, \bar{b})} \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(\bar{b}-m) m! \, (\bar{a}+m)} e^{-(\bar{a}+m)(\bar{\alpha}x)^{-\bar{\beta}}} \quad (٣٩)$$

٤.١ Asymptotic Variance Covariance Matrix

The asymptotic variance covariance matrix of the parameters α, β, a and $b$is the inverted of the fisher information matrix I which is given as follows:

$$I = \begin{bmatrix} I_{\alpha\alpha} & I_{\alpha\beta} & I_{\alpha a} & I_{\alpha b} \\ I_{\beta\alpha} & I_{\beta\beta} & I_{\beta a} & I_{\beta b} \\ I_{a\alpha} & I_{a\beta} & I_{aa} & I_{ab} \\ I_{b\alpha} & I_{b\beta} & I_{ba} & I_{bb} \end{bmatrix}, \qquad (٤٠)$$

where $I_{ij} = -E\left[\frac{\partial^2 \ln L}{\partial \tau_i \tau_j}\right], \tau = [\alpha, \beta, a, b], i, j = 1,2,3,4.$

٥.Simulation Study

In this section The Bayesian estimators (under the complete linear exponential (LINEX), general entropy and square loss functions) and maximum likelihood estimator to the unknown parameters α,β,a and b will be introduced in the simulation study by using Mathcad program in the case of complete data. A simulation study of size ١٠٠٠ is taken and three sample sizes are taken: n=٢٠, ٣٠, ٣٠ and ١٠٠. The result are displayed in table (١)

Table (١)

N	parameter	M.L	Squared	LINEX	General Entropy
٢٠	a	٠.٢٣٩	٠.١٢٢	٠.١٨٢	٠.١٤٨
	b	٠.٧٠٦	٠.٧٥٥	٠.٦٧٢	٠.٧٦٣
	□	٠.٣١٥	٠.٢٤٨	٠.٢٠١	٠.٢٩١
	□	٠.٤٨٤	٠.٤٣٩	٠.٣٥١	٠.٣٠١
٣٠	a	٠.٢٠٢	٠.١١٥	٠.١٠٩	٠.١١٢
	b	٠.٥٦٦	٠.٦٧٤	٠.٥٥٣	٠.٧١٢
	□	٠.٢٣٩	٠.١٤٥	٠.١٩٨	٠.٢٤٥
	□	٠.٣٧٦	٠.٣٣٤	٠.٢٧٦	٠.٢٨٥

٥٠	a	٠.١٢٤	٠.١٠٦	٠.٠٨٤	٠.١٠٤
	b	٠.٥٤٥	٠.٤٩٠	٠.٣٨١	٠.٤٥٦
	□	٠.٢٠٠	٠.١٩٦	٠.١٤٢	٠.٢٠٢
	□	٠.٣٣٣	٠.٢٦٥	٠.١١١	٠.١٩٩
١٠٠	a	٠.١٧٧	٠.٠٧٦	٠.٠٥٥	٠.٠٨٣
	b	٠.٣٧٤	٠.٢١٧	٠.٣٠١	٠.٣٥١
	□	٠.١٧٩	٠.١٠٦	٠.٠٣٢	٠.١٣٦
	□	٠.٢٠٣	٠.١٦٩	٠.٠٩٣	٠.٠٩٨

٦. Comments

- The Baysian estimation of Linex loss function better than the other loss functions
- Sometimes the LINEX gave to us results are closed to MLE
- When the sample size became bigger the result became best

٧. Conclusion

In this paper, the Bayesian estimationand maximum likelihood estimation to the parameters of Beta Inverse-Weibull distribution was presented. Bayes estimators are obtained using squared, LINEX and general entropy, and the maximum likelihood estimation to the same parameters are obtained, then we make a comparison between both of them. We expect that the new Beta Inverse-Weibull distribution will be useful for the practitioners.

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