التقدير البيزي لمعلمة الوزن في توزيع بيتا ـ معكوس وايبل باستخدام دوال الخسارة غير المتماثلة

Bayesian Estimation of the Scale Parameter Under Asymmetric Loss Functions to the Beta Inverse Weibull Distribution

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يعتبر توزيع معكوس وايبل من أهم التوزيعات الاحتمالية الخاصة بدراسة نظريات الصلاحية والتي تستخدم بكثرة في العلوم التطبيقية والفيزيائية والذي ظهر علي يد كيللر وكاماث ١٩٨٢ لاعتبارات فيزيقية خاصة بدراسة أعطال محركات الديزل وكتعميم لهذا التوزيع لإعطائه مرونة أكثر تم اشتقاق توزيع بيتا – معكوس وايبل باستخدام لوغاريتم لتوزيع بيتا علي يد خان ٢٠١٠ وانضم إلى التوزيعات التي تستخدم لدراسة أعطال الأجزاء الساكنة في الأجهزة والمحركات والفشل الحيوي

ويهدف هذا البحث لدراسة خواص التوزيع وتقدير معلمة الوزن باستخدام الأسلوب البيزي كأحد الأساليب الحديثة

* تم استنتاج خواص التوزيع مثل العزوم والدالة المولدة للعزوم ودالة الصلاحية والفشل

* تم استنتاج بعض التوزيعات التي تمثل حالات خاصة من توزيع بيتا معكوس وايبل عند قيم مختلفة لمعالمه

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* تم استخدام طريقة بيز لتقدير باعتبارها من الطرق الحديثة التي تتعامل مع المعالم على أنها متغيرات عشوائية تتبع توزيعات إحصائية احتمالية فيما تسمي التوزيعات القبلية وتم تقدير معلمة الوزن باستخدام دوال الخسارة الغير متماثلة

Abstract

The inverse Weibull distribution (IW) has the ability to model failure rates which are quite common in reliability and biological studies. A generalization of inverse Weibull distribution referred to as the Beta inverse Weibull distribution (BIW) which is generated from the logit of beta random variable was introduced as a life time distribution to give more flexible than the invers Wibull distribution. In this paper, the Bayesian approaches used to obtain the estimators of the scale parameter α for BIW distribution. Bayes estimators are derived by considering non-informative and informative prior distributions based on LINEX, general entropy and binary loss functions.

Keywords: Beta inverse Weibull distribution, Bayesian Estimation, scale parameter, loss function, inverted gamma distribution.

١. Introduction

The inverse Weibull distribution has received considerable attention in the literature. Keller and Kamath (١٩٨٢) studied the shapes of the density and failure(hazard) rate functions for the basic inverse model and Keller et al. (١٩٨٥) used the model for the reliability analysis of commercial vehicle engines. Erto (١٩٨٩) introduced further properties and identification of the model. Calabria and Pulcini (١٩٨٩،١٩٩٠) dealt with parameter estimation of the model. Jiang and Murthy (١٩٩٩) considered Weibull and inverse Weibull mixture models with negative weights. Also,

Drapella (١٩٩٣) and Jiang et al. (٢٠٠١) introduced graphical plotting techniques, known as the inverse Weibull probability paper (IWPP) plot and the Weibull probability paper (WPP) plot to determine the suitability of the Weibull and the inverse Weibull models for fitting a given data set. They showed that if IWPP plot is roughly a straight line, then the inverse Weibull model may be used to fit the given data set. Similarly if WPP plot is roughly a straight line, then the Weibull model may be used to fit the data set concerned. Khan et al. (٢٠٠٨) have discussed the classical statistical properties of IW distribution .Kim et al. (٢٠١٢) have derived the non-informative matching and references priors for the parameters of IW distribution.

In this paper, the focus of our attention is concentrated on the generalization of the inverse Weibull distribution referred to as the Beta inverse Weibull distribution which is generated from the logit of a beta random variable. Generalized beta distributions have been widely studied in statistics and numerous authors have developed various classes of these distributions. One major benefit of the class of beta generalized distributions is its ability of fitting of skewed data that cannot be properly fitted by existing distributions.

generalized class of probability distributions discussed by Eugene et al. (٢٠٠٢). Let $G(y)$be the cumulative distribution function (cdf) of a random variable Y . The cdf's for a generalized class of distributions for the random variable Y , defined by Eugene et al. (٢٠٠٢) as the logit of beta random variable, is given as:

$$F(y) = I_{G(y)}(a,b) \qquad a > 0 \quad and \ b > 0. \qquad (١)$$

Where

$$I_{G(y)}(a,b) = \frac{B_{G(y)}(a,b)}{B(a,b)} \quad \text{and} \quad B_{G(y)}(a,b) = \int_0^{G(y)} t^{a-1}(1-t)^{b-1}\,dt. \quad (٢)$$

Eugene et al. (٢٠٠٢) introduced the Beta normal distribution by taking $G(y)$ to be the cdf of the normal distribution. The only properties of the beta normal distribution known are some first moments derived by Eugene et al. (٢٠٠٢) and some more general moment expressions derived by Gupta and Nadarajah (٢٠٠٤). More recently, Nadarajah and Kotz (٢٠٠٤) were able to provide closed form expressions for the moments, the asymptotic distribution of the extreme order statistics and the estimation procedure for the beta Gumbel distribution. Cordeiro et al. (٢٠١١) proposed the Beta generalized exponential (BGE) distribution which generalizes the beta exponential distribution discussed by Nadarajah and Kotz (٢٠٠٥) and the generalized exponential (also named exponentiated exponential) distribution introduced by Gupta and Kundu (١٩٩٩). They provided a comprehensive mathematical treatment of BGE distribution with the hope that this generalization might attract wider applications in reliability and biology.

The BIW distribution was first introduced by Khan (٢٠١٠) as a new reliability model by taking $G(y)$ to be the cdf of the inverse Weibull distribution. The basic theoretical properties of the distribution are discussed in the paper , including the r^{th} moment, r^{th}inverse integer moment and maximum likelihood estimation, mean, variance, coefficient of variation, coefficient of skewness and coefficient of kurtosis for BIW distribution are presented mathematically.

The rest of the paper is organized as follows. The BIW distribution and its properties are given in Section ٢ . In section ٣ Bayes estimators of the scale parameter for BIW are derived by considering non-informative and informative prior distributions based on LINEX, general entropy and binary loss functions. Some concluding remarks and some future research proposals are given in Section٤.

٢. The Beta Inverse Weibull Distribution

٢,١. Probability Density Function

In this section, we present some theoretical properties for the Beta inverse Weibull distribution including the probability density function (pdf),cumulative distribution function (cdf), shape of the pdf, reliability and hazard functions, moments and the moment generating functions.

The probability density function (pdf) of the Beta inverse Weibull distribution is given by as follows:

$$f(y)=\frac{\beta\alpha^{-\beta}}{\beta(a,b)}y^{-(\beta+1)}e^{-a(\alpha y)^{-\beta}}\left[1-e^{-(\alpha y)^{-\beta}}\right]^{b-1}, \text{ for}$$

$$y\geq 0,\alpha>0,\beta>0,a>0 \text{ and } b>0. \quad (٣)$$

The pdf of the BIW distribution in (٣) also can be found by using this transformation $Y=\frac{[-\log_e(x)]^{-\frac{1}{\beta}}}{\alpha}$ where X is a random variable that follows a beta distribution with parameters *a* and *b* as follows:

$$Y=\frac{[-\log_e(x)]^{-\frac{1}{\beta}}}{\alpha}$$

$$g_Y(y)=\frac{\beta\alpha^{-\beta}}{\beta(a,b)}y^{-(\beta+1)}e^{-a(\alpha y)^{-\beta}}\left[1-e^{-(\alpha y)^{-\beta}}\right]^{b-1}, for y\geq 0,\alpha>0,\beta>0,$$

$a>0$, and $b>0$.

Special Cases

In equation (٣) if we take one or more parameters specific values, this results in other distributions as follows:

If β=١ , then the pdf (٣) becomes identically to the pdf of Beta inverse exponential distribution (BIE), i.e.,

$$f(y)=\frac{1}{\beta(a,b)}\frac{1}{\alpha y^2}e^{-\frac{a}{\alpha y}}\left[1-e^{-\frac{1}{\alpha y}}\right]^{b-1}, \text{for } y\geq 0,\alpha>0,a>0,b>0. \ (٤)$$

If β=٢ , then the pdf (٣) becomes identically to the pdf of Beta inverse Rayleighdistribution (BIR) , i.e.,

$$f(y)=\frac{1}{\beta(a,b)}\frac{2}{\alpha^2y^3}e^{-\frac{a}{(\alpha y)^2}}\left[1-e^{-\frac{1}{(\alpha y)^2}}\right]^{b-1}, \text{for } y\geq 0,\alpha>0,a>0,b>0. \quad (٥)$$

For a=١,b=١ and β=١ the distribution in (٣) reduces to standard inverse Exponential distribution (SIE), i.e.,

$$f(y) = \frac{1}{\alpha y^2} e^{-\frac{1}{\alpha y}} \quad , \quad \text{for } y \geq 0 \,, \alpha > 0. \qquad (٦)$$

for a = ١, b = ١, and β =٢ the distribution in (٣) reduces to standard inverse Rayleigh distribution (SIR), i.e.,

$$f(y) = \frac{2}{\alpha^2 y^3} e^{-\frac{1}{(\alpha y)^2}}, \quad \text{for } y \geq 0 \,, \alpha > 0. \qquad (٧)$$

If a = ١and b = ١, then (٣) reduces to the two parameter Inverse Weibull distribution(IW), i.e.,

$$f(y) = \beta \alpha^{-\beta} y^{-(\beta+1)} e^{-(\alpha y)^{-\beta}}, \quad \text{for } y \geq 0 \,, \alpha > 0, \beta > 0. \qquad (٨)$$

Table ١. Beta inverse Weibull type distribution

Distribution	BIW(α,β,a,b)			
	β	α	a	b
BIE	١	α	a	b
BIR	٢	α	a	b
SIE	١	α	١	١
SIR	٢	α	١	١
IW$(\alpha,\beta)\beta\alpha$ ١	١			

Cumulative Distribution Function

The Cumulative Distribution Function (cdf) for the Beta inverse Weibull distribution is given from (١) as follows:

$F(y) = \frac{1}{B(a,b)}\int_0^{G(y)}[G(u)]^{a-1}[1-G(u)]^{b-1}\,dG(u)$, and

$G(y) = e^{(-(\alpha y)^{-\beta})}, y \geq 0, \alpha > 0, \beta > 0.$

Let $G(u) = t$

$$F(y) = \frac{\beta\alpha^{-\beta}}{\beta(a,b)}\int_0^y t^{-(\beta+1)}e^{-a(\alpha t)^{-\beta}}\left[1-e^{-(\alpha t)^{-\beta}}\right]^{b-1}dt$$

Let $x = (\alpha t)^{-\beta}$ (٩)

$F(y) = \frac{1}{\beta(a,b)}\int_{(\alpha y)^{-\beta}}^{\infty} e^{-ax}[1-e^{-x}]^{b-1}dx$, for

$y \geq 0, \alpha > 0, \beta > 0, a > 0, b > 0.$ (١٠)

If $b > 0$ and b is non integer, real number , then

we can use the formula $(1-w)^{b-1} = \sum_{j=0}^{\infty}\frac{(-1)^j\Gamma(b)w^j}{\Gamma(b-j)j!}$ in (١٠) ,

$F(y) = \frac{\Gamma(b)}{\beta(a,b)}\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma(b-j)j!\,(a+j)}e^{-(a+j)(\alpha y)^{-\beta}}$. (١١)

This is an alternative representation of the cdf of the Beta inverse Weibull distribution in terms of an infinite series.

٢,٢. Reliability And Hazard Functions

The reliability function for any probability distribution is given as:

$R(y) = 1 - F(y).$ (١٢)

For BIW distribution the reliability function is:

$$R(y) = 1 - \frac{1}{\beta(a,b)} \int_{(\alpha y)^{-\beta}}^{\infty} e^{-ax}[1 - e^{-x}]^{b-1} dx. \quad (١٣)$$

If $b > 0$ and b is non integer, real number , then

$$R(y) = 1 - \frac{\Gamma(b)}{\beta(a,b)} \sum_{j=0}^{\infty} \frac{(-1)^j}{\Gamma(b-j)j!\,(a+j)} e^{-(a+j)(\alpha y)^{-\beta}} \quad (١٤)$$

The hazard rate function for any probability distribution is given as:

$$h(y) = \frac{f(y)}{1-F(y)} \quad (١٥)$$

For BIW distribution the hazard rate function is

$$h(y) = \frac{\beta\alpha^{-\beta} y^{-(\beta+1)} e^{-a(\alpha y)^{-\beta}} \left[1-e^{-(\alpha y)^{-\beta}}\right]^{b-1}}{\beta(a,b) - \int_{(\alpha y)^{-\beta}}^{\infty} e^{-ax}[1-e^{-x}]^{b-1} dx}. \quad (١٦)$$

If $b > 0$ and b is non integer, real number , then

$$h(y) = \frac{\beta\alpha^{-\beta} y^{-(\beta+1)} e^{-a(\alpha y)^{-\beta}} \left[1-e^{-(\alpha y)^{-\beta}}\right]^{b-1}}{\beta(a,b) - \Gamma(b) \sum_{j=0}^{\infty} \frac{(-1)^j}{\Gamma(b-j)j!\,(a+j)} e^{-(a+j)(\alpha y)^{-\beta}}}. \quad (١٧)$$

The function (١٦) is important in characterizing the phenomena of the distribution.

٢,٣. Moments

To derive moments of BIW distribution, consider the density given in (٣). The r^{th} central moment of the distribution is given by

$$\mu_r^{'} = E(y^r) = \int_0^{\infty} y^r f(y) dy \quad (١٨)$$

$$= \frac{\beta\alpha^{-\beta}}{\beta(a,b)} \int_0^{\infty} y^{c-1} e^{-a(\alpha y)^{-\beta}} \left[1 - e^{-(\alpha y)^{-\beta}}\right]^{b-1} dy,$$

Where$c = r - \beta$.

$$E(y^r) = \frac{\alpha^{-r}}{\beta(a,b)} \int_0^\infty x^{k-1} e^{-ax} [1 - e^{-x}]^{b-1} dx \qquad (١٩)$$

Where $k = \frac{-c}{\beta} = 1 - \frac{r}{\beta}$.

For positive integer b,

$$E(y^r) = \frac{\alpha^{-r}}{\beta(a,b)} \sum_{j=0}^{b-1} \frac{(-1)^j (b-1)!}{(b-j-1)!\ \ j!} \int_0^\infty x^{k-1} e^{-(a+j)x} dx$$

$$E(y^r) \ = \frac{\alpha^{-r}}{\beta(a,b)} \Gamma(k) \sum_{j=0}^{b-1} \frac{(-1)^j (b-1)!}{(b-j-1)!\ j!} \left(\frac{1}{w}\right)^k, \qquad (٢٠)$$

where $w = (a+j)x$, then

for c > ٠ and b > ٠,

$$E(y^r) = \frac{\alpha^{-r}}{\beta(a,b)} \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(b)}{\Gamma(b-j) j!} \int_0^\infty x^{k-1} e^{-(a+j)x} dx$$

$$= \frac{\alpha^{-r}}{\beta(a,b)} \Gamma(k) \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(b)}{\Gamma(b-j) j!} \left(\frac{1}{w}\right)^k. \qquad (٢١)$$

٢,٤. Generating Function

We derive the moment generating function and characteristic function for a BIW distribution as follows:

$$M_y(t) = E(e^{ty}) = \int_0^\infty e^{ty} f(y) dy \qquad (٢٢)$$

$$= \frac{\beta \alpha^{-\beta}}{\beta(a,b)} \int_0^\infty e^{ty} y^{-(\beta+1)} e^{-a(\alpha y)^{-\beta}} \left[1 - e^{-(\alpha y)^{-\beta}}\right]^{b-1} dy.$$

By using (Taylor's series expansion of e^{ty} about zero)

$$e^{ty} = \sum_{j=0}^{\infty} \frac{(ty)^j}{j!}$$

$$M_y(t) = \frac{\beta\alpha^{-\beta}}{\beta(a,b)} \int_0^{\infty} \sum_{j=0}^{\infty} \frac{t^j}{j!} y^{-(\beta+1-j)} e^{-a(\alpha y)^{-\beta}} \left[1 - e^{-(\alpha y)^{-\beta}}\right]^{b-1} dy.$$

Using transformation in (٩) , then

$$M_y(t) = \frac{1}{\beta(a,b)} \sum_{j=0}^{\infty} \frac{t^j}{j!} \alpha^{-j} \int_0^{\infty} x^{-\frac{j}{\beta}} e^{-ax} [1 - e^{-x}]^{b-1} dx.$$

Let $w = e^{-x}$, then

$$M_y(t) = \frac{1}{\beta(a,b)} \int_0^1 \left[e^{(\ln w^{-1})^{-\frac{1}{\beta}}}\right]^{\frac{t}{\alpha}} w^{a-1} [1 - w]^{b-1} dw. \qquad (٢٣)$$

It follows from the above that the characteristic function (CF) of the Beta inverse Weibull distribution is given by:

$$\Phi_y(t) = E\left(e^{ity}\right) \qquad (٢٤)$$

$$= \frac{1}{\beta(a,b)} \int_0^1 \left[e^{(\ln w^{-1})^{-\frac{1}{\beta}}}\right]^{\frac{it}{\alpha}} w^{a-1} [1 - w]^{b-1} dw \qquad (٢٥)$$

٣. Bayesian Estimation of The Scale parameter

In Bayesian estimation and prediction problems, the performance of estimators depends on the form of the prior distribution and the loss function. Choice of loss function is an essential part, since, there is no specific analytical procedure to identify the appropriate loss function to be used; in most of the studies on estimation and prediction problems, authors for convenience consider the underlying loss function to be squared error which is symmetric in nature. However, in-discriminate use of squared error loss function is not appropriate particularly in the cases, where the losses are not symmetric. From this viewpoint, Varian (١٩٧٥) proposed the asymmetric LINEX loss function, and Zellner (١٩٨٦) extensively discussed its properties. Despite the flexibility of the linex loss function for the estimation of a location parameter, it appears not to be suitable for the estimation of scale parameters and other quantities. For these reasons Basu and Ibrahimi (١٩٩١) defined a modified linex loss function. Calabria and Pulcini (١٩٩٤) proposed another alternative to the modified linex loss function named general entropy loss function.

In this section we find the Bayesian estimator of scale parameter α for BIW distribution when the other parameters β, a, b are known using non-informative and informative prior distributions based on LINEX, general entropy and binary loss functions. Generally, A loss function $l(\hat{\alpha},\alpha)$ represents losses incurred when we estimate the parameter α by $\hat{\alpha}$. The linex loss function is defined as;

$$l(\hat{\alpha},\alpha) = d[e^{[c(\hat{\alpha}-\alpha)]} - c(\hat{\alpha}-\alpha) - 1], \qquad (٢٦)$$

with two parameters $d > 0$, $c \neq 0$, where, $d$is the scale of the loss function and c determines its shape . The Bayes estimator under the linex loss function (٢٦) obtained by minimizing the posterior expected loss or posterior risk (average loss)as,

$$\hat{\alpha} = -\frac{1}{c} lin[E_{\alpha}(e^{-c\alpha})]. \qquad (٢٧)$$

General entropy loss function is defined as,

$$l(\hat{\alpha},\alpha) \propto \left(\frac{\hat{\alpha}}{\alpha}\right)^{q} - q\ln\left(\frac{\hat{\alpha}}{\alpha}\right) - 1 \qquad (٢٨)$$

which has a minimum at $\hat{\alpha}=\alpha$. This loss is a generalization of the entropy lossfunction used by several authors taking the shape parameter $q = 1$. The more general version (٢٨) allows different shapes of loss function when $q > 0$ and for $\hat{\alpha}=\alpha$, i.e.a positive error causes more serious consequences than a negative error. The Bayes estimator of α under the general entropy loss will be,

$$\hat{\alpha} = \left[E_{\alpha}\{\alpha^{-q}\}\right]^{-\frac{1}{q}}. \qquad (٢٩)$$

Binary (٠،١) loss function is defined as,

$$l(\hat{\alpha},\alpha) = \begin{cases} 1 & |\hat{\alpha}-\alpha| \geq \varepsilon \\ 0 & |\hat{\alpha}-\alpha| < \varepsilon \end{cases}. \qquad (٣٠)$$

The Bayes estimator under the binary (٠،١) loss function (٣٠) obtained by finding the mode of the posterior distribution of α which is minimizing the posterior expected loss , i.e. we differentiate the $\pi(\alpha|\underline{x})$ to α and equating it with Zero to evaluating the estimator of α as,

$$\frac{d\,\pi(\alpha|\underline{x})}{d\alpha} = 0 \qquad (٣١)$$

٣,١. Bayesian Estimation of The Scale Parameter Based on Non-Informative Prior Distribution

In Bayesian analysis, the parameter of interest is considered to be a random variable having some prior distribution. The selection of prior distribution is often based on the type of prior information available to us. When we have little or no information about the parameter, a non-informative prior should be used. The important feature of this prior is that it is not affected by the restriction of the parameter space.In this section we have studied the posterior distribution ofα using non-informative prior distribution as under (which used in Bayesian estimation) :

$$\pi(\alpha) = \frac{1}{\alpha};\ \alpha > 0. \qquad (٣٢)$$

Consider a random sample of size n consisting of values $x_1, x_2, \dots, x_n$from the density function defined in (٣) , then the likelihood function for α is given by:

$$L(\alpha) =$$

$$\left[\frac{1}{\beta(a,b)}\right]^n \beta^n (\alpha^{-\beta})^n \prod_{i=1}^{n} [x_i^{-(\beta+1)}] \left(e^{-\alpha \sum_{i=1}^{n} (\alpha x_i)^{-\beta}}\right) \prod_{i=1}^{n} \left[1 - e^{-(\alpha x_i)^{-\beta}}\right]^{b-1} \qquad (٣٣)$$

The posterior distribution of α given $x_1, x_2, \dots, x_n$ is obtained as,

$$\pi(\alpha|\underline{x}) = \frac{L(\alpha)\pi(\alpha)}{\int_0^\infty L(\alpha)\pi(\alpha)\,d\alpha} = \frac{I}{I_1}. \qquad (٣٤)$$

Substituting $L(\alpha)$ and $\pi(\alpha)$ from Equations (٣٣) and (٣٢), respectively, in Equation (٣٤), we get, after simplification, the posterior distribution as

$$\pi(\alpha|\underline{x}) = A_1 \alpha^{-(n\beta+1)} \sum_{j=0}^{\infty} u^n \left[e^{-z\alpha^{-\beta}}\right]^u;\ \alpha > 0, \qquad (٣٥)$$

where $A_1 = \frac{\beta z^n}{\Gamma(n)}, u = a + j\ ,\ z = \sum_{i=1}^{n} x_i^{-\beta}$.

٣,١,١. Bayes Estimator Under LINEX Loss Function

The Bayes estimator of scale parameter α for BIW distribution under LINEX loss function is obtained by using (٣٥) and (٢٧) then,

$$\hat{\alpha} = -\frac{1}{c} lin\left[A_1 \sum_{j,k=0}^{\infty} \frac{(-z)^k}{k\,!} u^{n+k} \int_0^{\infty} \alpha^{-\beta(n+k)-1} e^{-c\alpha} d\alpha\right]. \qquad (٣٦)$$

٣,١,٢. Bayes Estimator Under General Entropy Loss Function

The Bayes estimator of scale parameter α for BIW distribution under general entropy loss function is obtained by using (٣٥) and (٢٩) then,

$$\hat{\alpha} = \left[\frac{\Gamma(v)}{\Gamma(n)} z^{n-v} \sum_{j=0}^{\infty} u^{n-v}\right]^{-\frac{1}{q}}, \qquad (٣٧)$$

where $v = n + \frac{q}{\beta}$

٣,١,٣. Bayes Estimator Under Binary (٠،١) Loss Function

By using (٣٥) in (٣١) then,

$$z\beta\hat{\alpha}^{-(n\beta+\beta+2)} \sum_{j=0}^{\infty} u^{n+1}\left[e^{-(uz)\hat{\alpha}^{-\beta}}\right] - (n\beta+1)\hat{\alpha}^{-(n\beta+2)} \sum_{j=0}^{\infty} u^{n}\left[e^{-(uz)\hat{\alpha}^{-\beta}}\right] = 0, \qquad (٣٨)$$

the Bayes estimator of scale parameter α for BIW distribution under binary (٠،١) loss function is obtained by solving (٣٨) with respect to $\hat{\alpha}$.

٣,٢. Bayesian Estimation of The Scale Parameter Based on Informative Prior Distribution

Assume that the scale parameters α has informative prior distribution given by the inverted-gamma density as follow:

$$\pi(\alpha) = \frac{\lambda^r}{\Gamma(r)} \alpha^{-r-1} e^{-\frac{\lambda}{\alpha}} \;\; ; \;\; \alpha > 0 \qquad (٣٩)$$

Substituting $L(\alpha)$ and $\pi(\alpha)$ from Equations (٣٣) and (٣٩), respectively, in Equation (٣٤), we get, after simplification, the posterior distribution as,

$$\pi(\alpha|\underline{x}) = A_2 \alpha^{-(n\beta+r+1)} e^{-\gamma\alpha^{-\beta} - \frac{\lambda}{\alpha}}; \;\; \alpha > 0 \;, \qquad (٤٠)$$

where $A_2 = \beta \sum_{j,k=0}^{\infty} \frac{k!\, \gamma^d}{(-\lambda)^k \Gamma(d)}$, $\gamma = \left[(a+j)(\sum_{i=1}^{n} x_i^{-\beta})\right]$, $d = n + \frac{r+k}{\beta}$.

٣,٢,١. Bayes Estimator Under LINEX Loss Function

The Bayes estimator of scale parameter α for BIW distribution under LINEX loss function is obtained by using (٤٠) in (٢٧) then,

$$\hat{\alpha} = -\frac{1}{c} lin \left[\beta \sum_{j,k,m,l=0}^{\infty} \frac{k!\, \gamma^d}{(-\lambda)^k \Gamma(d)} \frac{(-\gamma)^m}{m!} \frac{(-c)^l}{l!} \frac{\Gamma(d_2)}{\lambda^{d_2}}\right]. \qquad (٤١)$$

Where $\gamma = \left[(a+j)(\sum_{i=1}^{n} x_i^{-\beta})\right]$, $d = n + \frac{r+k}{\beta}$, $d_2 = \beta(n+m) + r - l$

٣,٢,٢. Bayes Estimator Under General Entropy Loss Function

The Bayes estimator of scale parameter α for BIW distribution under general entropy loss function is obtained by using (٤٠) in (٢٩) then,

$$\hat{\alpha} = \left[\sum_{j,k,m=0}^{\infty} \frac{k!\,\gamma^{d}}{(-\lambda)^{k}\,\Gamma(d)} \frac{(-\lambda)^{m}}{m!} \frac{\Gamma(d_3)}{\gamma^{d_3}}\right]^{-\frac{1}{q}} \quad (٤٢)$$

Where $d_3 = n + \frac{1}{\beta}(q + r + m)$

٣,٢,٣. Bayes Estimator Under Binary Loss Function

By using (٤٠) in (٣١) then,

$$\hat{\alpha}^{-(n\beta+r+1)}\left[\gamma\beta\hat{\alpha}^{-(\beta+1)} + \lambda\hat{\alpha}^{-2}\right]e^{-\gamma\hat{\alpha}^{-\beta}-\frac{\lambda}{\hat{\alpha}}} - (n\beta + r + 1)\hat{\alpha}^{-(n\beta+r+2)}e^{-\gamma\hat{\alpha}^{-\beta}-\frac{\lambda}{\hat{\alpha}}} = 0, \quad (٤٣)$$

we get, after simplification

$$\gamma\beta\hat{\alpha}^{-(\beta+1)} + \lambda\hat{\alpha}^{-2} - (n\beta + r + 1)\hat{\alpha}^{-1} = 0, (٤٤)$$

the Bayes estimator of scale parameter α for BIW distribution under binary (٠, ١) loss function is obtained by solving (٤٤) with respect to $\hat{\alpha}$.

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