طريقة جديدة لتقدير معالم جهد ـ قوة الصلاحية لتوزيع ريلاي المعمم

دكتور/ طارق محمد شمس الدين(*)
دكتور/ حمدى محمد سالم محمد(**)

ملخص البحث

مقدمــة:

يستخدم توزيع ريلاي Rayleigh distribution على نطاق واسع في العديد من المجالات كالعلوم الاجتماعية والطبيعية وكذلك في المجالات الطبية، فعلى سبيل المثال، يستخدم في دراسة أنواع مختلفة من الإشعاع وتوليد طاقة الرياح.

وفى السنوات الأخيرة تم تعميم توزيعات متنوعة مثل التوزيع الاسى وتوزيعات باريتو وذلك بإضافة معلمة شكل لدالة التوزيع التراكمية.

وهناك العديد من المؤلفين الذين تناولوا هذا النوع من الدراسات. علي سبيل المثال، Mudholkar and Hutson [4] و Gupta and Kundu [13] و Surles and Padgett [7].

والصفة المشتركة التي تجمع هذا النوع من التوزيعات هى أن دالة التوزيع التراكمية يمكن كتابتها على الشكل $F(x) = [G(x)]^{\alpha}$ حيث $G(.)$ هى دالة التوزيع التراكمية للتوزيع الأصلي و $\alpha > 0$ تشير إلى معلمة الشكل المضافة كأس للدلة $G(.)$.

(*) الأستاذ المساعد بقسم الإحصاء ، كلية التجارة بنين جامعة الأزهر
(**) المدرس بقسم الإحصاء، كلية التجارة بنين جامعة الأزهر

ويذكر أن توزيع Rayleigh ذو المعلمتين (scale and shape) يطلق عليه توزيع بيير من النوع العاشر Burr type X .

وقام بدراسة توزيع ريلاي المعمم Generalized Rayleigh distribution كل من [5] Sartawi and Abu-Salih و [16, 17] Jaheen و Ahmad et al. [8] في حالة (scale parameter equals one) .

وقد لاحظ [14] Surles and Pudgett أن Generalized Rayleigh distribution ذو المعلمتين يمكن استخدامه بكفاءة لدراسة نماذج صلاحية ضغط - قوة ونماذج اختبارات الحياة.

ودالة كثافة الاحتمال لتوزيع ريلاي المعمم -The exponentiated Rayleigh distribution على الشكل التالى:

$$f(x) = \frac{2\,x\,\theta}{\sigma^2} e^{-\left(\frac{x}{\sigma}\right)^2}\left[1 - e^{-\left(\frac{x}{\sigma}\right)^2}\right]^{\theta-1}; \quad x > 0 \quad ,\sigma > 0$$

ودالة التوزيع التراكمية هى :

$$F(x) = \left[1 - e^{-\left(\frac{x}{\sigma}\right)^2}\right]^{\theta} \quad ; x > 0 \quad ,\sigma > 0$$

هدف البحث

يهدف البحث الي تقدير معالم نموذج ضغط- قوة لتوزيع ريلاي المعمم عندما يكون كلا من الضغط Y والقوة X مستقلان على التوالى. وكذلك اشتقاق التوزيع الهامشى لمعلمة الصلاحية $R = P(Y < X)$ باستخدام طريقة التحويلات حيث تلعب هذه المعلمة دورا مهما في قياس أداء النظام.

والهدف الرئيسي لهذا البحث هو تطبيق النتائج النظرية التى تم التوصل إليها على بيانات واقعية للتأكد من كفاءة النموذج.

خطة البحث

يتألف البحث من ٤ مباحث.

مبحث ١ : ويشمل مقدمة للبحث مشتملة لبعض الدراسات السابقة ذات الصلة بالموضوع.

مبحث ٢ : تم تقدير معلمة الصلاحية باستخدام طريقة الإمكان الأكبر.

مبحث ٣ :استخدمت طريقة التحويلات لاشتقاق التوزيع الهامشي للمعلمة R .

مبحث ٤ :تم تطبيق النتائج التى تم التوصل اليها على بيانات واقعية للتأكد من كفاءة النموذج.

مبحث ٥ : عرض نتائج البحث.

A New Estimation Technique of Stress- Strength Reliability Parameter for Exponentiated-Rayleigh Distribution

Dr. Tarek M. Shams, Dr. Hamdy M. Salem
Department of Mathematical Statistic, Faculty of Commerce, Al-Azhar University

Abstract

This paper deals with the estimation of the stress-strength parameter $R = P(Y < X)$ when X and Y are independent random variables with different shape parameters. The maximum likelihood estimator of the unknown parameters can be obtained in explicit form. We obtain the asymptotic distribution of the maximum likelihood estimator of R. Transformation Techniques will be used to drive distribution of.We provide illustration with an application to a real data set.

Keywords: Exponentiated-Rayleigh Distribution; Maximum Likelihood Estimator; fisher information matrix; Asymptotic-distribution.

1. Introduction

The Rayleigh distribution is widely used to model events that occur in different fields such as medicine, social and natural sciences. For instance, it is used in the study of various types of radiation, such as sound and light measurements. It is also used as a model for wind speed and is often applied to wind-driven electrical generation. In recent years, several standard life time distributions have been generalized via exponentiation. Examples of such exponentiated distributions are the exponentiated Weibull family, the exponentiated exponential, and the exponentiated (generalized) Rayleigh and the exponentiated Pareto family of distributions.

Among the authors who have considered the exponentiated distributions are Mudholkar and Hutson [4], Gupta and Kundu [13], Surles and Padgett [7] and Kundu and Gupta [2, 3].A common feature in families of exponentiated distributions is that the distribution function may be written as $F(x) = [G(x)]^{\alpha}$, where $G(.)$ is the distribution function of a corresponding non-generalized Distribution and $\alpha > 0$ denotes the generalization parameter. The generalized Rayleigh distributionis obtained by generalization of the Rayleigh distribution. It is also called the two parameter (scale and shape) Burr type ▯ distribution. The generalized Rayleigh density can be used to study skewed data set. The one parameter (scale parameter equals one) generalized Rayleigh distribution is studied by Sartawi and Abu-Salih [5], Jaheen [16, 17], Ahmad et al. [8], Raqab[10], and Surlesand Padgett [6]. Recently Surles and Pudgett [14] observed that the two parameters generalized Rayleigh distribution can be used quite effectively in modeling strength and life time data. Kundu and Raqab [1] used different methods to estimate the unknown parameters of the generalized Rayleigh. Raqab and Kundu [9] discuss several interesting properties of the Generalized Rayleigh distribution, Recently, Surles and Padgett (14) (see also Surles and Padgett, 15) introduced two parameter Burr Type X distribution and correctly named as the generalized Rayleigh distribution. Note that the two-parameter generalized Rayleigh distribution is a particular member of the exponentiated Weibull distribution, originally proposed by Mudholkar and Srivastava (11), see also Mudholkar et al. (12).

The exponentiated-Rayleigh Distribution has the following probability density function (pdf):

$$f(x) = \frac{2\,x\,\theta}{\sigma^2} e^{-\left(\frac{x}{\sigma}\right)^2}\left[1 - e^{-\left(\frac{x}{\sigma}\right)^2}\right]^{\theta-1} \quad ; \; x > 0 \quad , \sigma > 0 \quad , \theta > 0 \qquad (1)$$

From now on if a random variable ▯ has the pdf (1), then it will be denoted by Exponentiated-Rayleigh distribution, the corresponding cumulative distribution function (cdf) and hazard rate function (hrf) are:

$$F(x) = \left[1 - e^{-\left(\frac{x}{\sigma}\right)^2}\right]^{\theta} \quad ; \; x > 0 \quad , \sigma > 0 \quad , \theta > 0 \qquad (2)$$

and

$$H(x) = \frac{\frac{2x\theta}{\sigma^2} e^{-\left(\frac{x}{\sigma}\right)^2} \left[1 - e^{-\left(\frac{x}{\sigma}\right)^2}\right]^{\theta-1}}{1 - \left[1 - e^{-\left(\frac{x}{\sigma}\right)^2}\right]^{\theta}} \tag{3}$$

The main aim of this paper is to develop the inferential procedure of the stress-strength parameter $R = P(Y < X)$, when X and Y are independent ER(θ_1) and ER(θ_2), respectively, and drive the distribution of R via transformation method. Note that the stress-strength parameter plays an important role in the reliability Analysis. For example if X is the strength of a system which is subjected to stress Y , thenthe parameter R measures the system performance and it is very common in the context of Mechanical reliability of a system. Moreover, R provides the probability of a system failure, if the system fails whenever the applied stress is greater than its strength.

In this paper, the main aim is to consider maximum likelihood estimator by using extensive of applications to a real data set, the rest of the paper is organized as follows. In Section 2, we provide the maximum likelihood estimator of R "MLE of R". In Section 3 we discuss transformation techniques and drive the marginal distribution. Applications to a real data set to illustrate the results and data analysis are presented in Section 4, respectively. Finally, we conclude the paper in Section 5.

2. Maximum likelihood estimators

In this section, we consider the maximum likelihood estimators (MLEs) of R. Suppose that X and Y are two independent RVs with respective parameters θ_1 and θ_2 having PDFs $f_x(\cdot)$ and $f_y(\cdot)$. Then

$$R = P(Y < X) = \int_0^\infty f_x(x).F_y(y)\,dx$$

$$= \frac{2\theta_1}{\sigma^2}\int_0^\infty x e^{-\left(\frac{x}{\sigma}\right)^2}\left[1 - e^{-\left(\frac{x}{\sigma}\right)^2}\right]^{\theta_1+\theta_2-1} dx$$

$$= \theta_1\left[\frac{\left[1 - e^{-\left(\frac{x}{\sigma}\right)^2}\right]^{\theta_1+\theta_2}}{\theta_1+\theta_2}\right]_0^\infty$$

$$= \frac{\theta_1}{\theta_1+\theta_2}$$

The log-likelihood function for MLE of R and its asymptotic distribution reduces to

$$L(x;\theta_1,\theta_2,\sigma) = \left(\frac{2\theta_1}{\sigma^2}\right)^n \left(\frac{2\theta_1}{\sigma^2}\right)^m \sum_{i=1}^{n} x_i.\sum_{j=1}^{m} y_i\, e^{-\sum_{i=1}^{n}\left(\frac{x_i}{\sigma}\right)^2}.e^{-\sum_{i=1}^{n}\left(\frac{y_i}{\sigma}\right)^2}.$$

$$\prod_{i=1}^{n}\left[1 - e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]^{\theta_1-1}.\prod_{j=1}^{m}\left[1 - e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]^{\theta_2-1} \qquad (4)$$

And

$$\ln y_i - \sum_{i=1}^{n}\left(\frac{x_i}{\sigma}\right)^2 - \sum_{i=1}^{n}\left(\frac{y_i}{\sigma}\right)^2 + (\theta_1 - 1)\sum_{i=1}^{n}\ln\left[1 - e^{-\left(\frac{x_i}{\sigma}\right)^2}\right] + (\theta_2 - 1)\sum_{j=1}^{m} \square$$

(5)

The score vector, where the components corresponding to the parameters are given by differentiating (5).

$$\frac{\partial \ln L}{\partial \theta_1} = \frac{n}{\theta_1} + \sum_{i=1}^{n}\ln\left[1 - e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]$$

(6)

$$\frac{\partial \ln L}{\partial \theta_2} = \frac{m}{\theta_2} + \sum_{j=1}^{m}\ln\left[1 - e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]$$

(7)

And

$$\frac{\partial \ln L}{\partial \sigma^2} = \frac{-(n+m)}{\sigma^2} + \sum_{i=1}^{n}\left(\frac{x_i}{\sigma^2}\right)^2 + \sum_{i=1}^{n}\left(\frac{y_i}{\sigma^2}\right)^2 + (\theta_1 - 1)\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\sigma^2}\right)^2 e^{-\left(\frac{x_i}{\sigma}\right)^2}}{\left[1 - e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]}$$

$$+(\theta_2 - 1)\sum_{j=1}^{m}\frac{\left(\frac{y_i}{\sigma^2}\right)^2 e^{-\left(\frac{y_i}{\sigma}\right)^2}}{\left[1 - e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]}$$

(8)

The maximum likelihood estimates (MLEs) of the parameters are the solutions of the nonlinear equations, which are solved iteratively.

We can obtain $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\sigma}^2$ from the explicit forms as the following:

$$\hat{\theta}_1 = \frac{n}{-\sum_{i=1}^{n} \ln \left[1 - e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]^{\theta_1 - 1}} \qquad (9)$$

$$\hat{\theta}_2 = \frac{m}{-\sum_{j=1}^{m} \ln \left[1 - e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]^{\theta_2 - 1}} \qquad (10)$$

Put $g(\sigma^2) = \square^2$ in equation (8) we obtain the following:

$$g(\sigma^2) = (n+m)\left\{\left[\frac{n}{\sum_{i=1}^{n} \ln \left[1 - e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]} - 1\right] . \sum_{i=1}^{n} \frac{\left(\frac{x_i}{\sigma^2}\right)^2 e^{-\left(\frac{x_i}{\sigma}\right)^2}}{\left[1 - e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]} - \sum_{i=1}^{n} \left(\frac{x_i}{\sigma^2}\right)^2\right.$$

$$\left.\left[\frac{n}{\sum_{j=1}^{m} \ln \left[1 - e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]} - 1\right] . \sum_{j=1}^{m} \frac{\left(\frac{y_i}{\sigma^2}\right)^2 e^{-\left(\frac{y_i}{\sigma}\right)^2}}{\left[1 - e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]} - \sum_{j=1}^{m} \left(\frac{y_i}{\sigma^2}\right)^2\right\}^{-1}$$

Hence, obtained the estimate value of $\hat{\sigma}^2$ and replaced in (9), and (10), we can compute the estimate value of $\hat{\theta}_1$ and $\hat{\theta}_2$ and then the MLEs of R will be as the following:

$$\hat{R} = \frac{\hat{\theta}_1}{\hat{\theta}_1 + \hat{\theta}_2}$$

$$\hat{R} = \frac{\frac{n}{w}}{\frac{n}{w} + \frac{m}{v}} \tag{11}$$

where $w = -\sum_{i=1}^{n} \ln\left[1 - e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]^{\theta_1 - 1}$ and $v = -\sum_{j=1}^{m} \ln\left[1 - e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]^{\theta_2 - 1}$

The last equation can be used to construct transformation techniques as in the next section

The maximum likelihood estimates (MLEs) of the parameters are the solutions of the nonlinear equations $\nabla \ell = 0$, which are solved iteratively. The observed information matrix is

$$J_n(\theta_1, \theta_2, \sigma^2) = n \begin{pmatrix} J_{\theta_1\theta_1} & J_{\theta_1\theta_2} & J_{\theta_1\sigma^2} \\ J_{\theta_2\theta_1} & J_{\theta_2\theta_2} & J_{\theta_2\sigma^2} \\ J_{\sigma^2\theta_1} & J_{\sigma^2\theta_2} & J_{\sigma^2\sigma^2} \end{pmatrix}$$

whose elements are:

$$J_{\theta_1\theta_1} = -\frac{n}{\theta_1^{\,2}} J_{\theta_1\theta_2} = 0$$

$$J_{\theta_1\sigma^2}=\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\sigma^2}\right)^2 e^{-\left(\frac{x_i}{\sigma}\right)^2}}{\left[1-e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]}$$

$$J_{\theta_2\theta_2}=-\frac{m}{\theta_2{}^2}J_{\theta_2\sigma^2}=\sum_{j=1}^{m}\frac{\left(\frac{y_i}{\sigma^2}\right)^2 e^{-\left(\frac{y_i}{\sigma}\right)^2}}{\left[1-e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]}$$

$$J_{\sigma^2\sigma^2}=\frac{n+m}{(\sigma^2)^2}-2\sum_{i=1}^{n}\frac{x_i{}^2}{\left(\sigma^2\right)^3}-2\sum_{i=1}^{n}\frac{y_i{}^2}{(\sigma^2)^3}$$

$$+(\theta_1-1)\sum_{i=1}^{n}\frac{\left(1-e^{-\left(\frac{x_i}{\sigma}\right)^2}\right)\left[\left(\frac{x_i}{\sigma^2}\right)^4.e^{-\left(\frac{x_i}{\sigma}\right)^2}-2\frac{x_i{}^2}{(\sigma^2)^3}.e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]-\left(\left(\frac{x_i}{\sigma^2}\right)^2 e^{-\left(\frac{x_i}{\sigma}\right)^2}\right)^2}{\left[1-e^{-\left(\frac{x_i}{\sigma}\right)^2}\right]^2}$$

$$+(\theta_2-1)\sum_{j=1}^{m}\frac{\left(1-e^{-\left(\frac{y_i}{\sigma}\right)^2}\right)\left[\left(\frac{y_i}{\sigma^2}\right)^4.e^{-\left(\frac{y_i}{\sigma}\right)^2}-2\frac{y_i{}^2}{(\sigma^2)^3}.e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]-\left(\left(\frac{y_i}{\sigma^2}\right)^2 e^{-\left(\frac{y_i}{\sigma}\right)^2}\right)^2}{\left[1-e^{-\left(\frac{y_i}{\sigma}\right)^2}\right]^2}$$

3- Transformation Techniques

Given two independent coordinates w and v from normal distributions with zero mean and the same variance σ^2 the distance $w = \frac{n(1-r)}{z.r}$ and $v = \frac{m}{z}$ is distributed according to the exponentiated-Rayleigh distribution. w and v may be regarded as the velocity components of a particle moving in a plane.

To realize this we first write,

$$w = \frac{n(1-r)}{z.r} \qquad \text{and} \qquad v = \frac{m}{z}$$

The Jacobian of the transformation is obtained as follows:

$$J = \begin{bmatrix} \frac{\partial w}{\partial r} & \frac{\partial w}{\partial z} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{-n}{z.r^2} & \frac{-n(1-r)}{z^2.r} \\ 0 & \frac{-m}{z^2} \end{bmatrix} = \frac{n.m}{z^3.r^2}$$

$$|J| = \left|\frac{n.m}{z^3.r^2}\right| = \frac{n.m}{z^3.r^2}$$

The joint probability density function (pdf) of w and v written as the following:

$$f_{R,Z}(r,z) = f_{w,v}(w,v).|J|$$

$$= f(w).f(v).|J|$$

$$;0 < r < 1, 0 < z < \infty \qquad (12)$$

where

$$f(w) = \frac{\theta_1^n}{\Gamma(n)}\left(\frac{n(1-r)}{z.r}\right)^{n-1}.e^{\frac{-\theta_1.n(1-r)}{z.r}} \quad ; \frac{n(1-r)}{z.r} > 0 \quad ,$$

$$f(v) = \frac{\theta_2^m}{\Gamma(m)}\left(\frac{m}{z}\right)^{m-1} e^{-\theta_2\left(\frac{m}{z}\right)} \quad ; \quad \left(\frac{m}{z}\right) > 0 \ ,$$

$$\beta = \frac{\theta_2 m}{z} \quad \text{and} \quad \alpha = \frac{\theta_1.n(1-r)}{z.r}$$

The marginal probability density function of r can be written as:

$$h(r) = \int_0^\infty f_{R,Z}(r,z)\,dz$$

$$h(r) = \beta_{(n,m)}^{-1}\left(\frac{\theta_2 m}{\theta_1 n}\right)^m.r^{m-1}(1-r)^{n-1}\left[1 - r\left(1 - \frac{\theta_2 m}{\theta_1 n}\right)\right]^{-(n+m)} \quad ;0 < r < 1$$

which seems like beta Distribution with the following property

$$E\left(r^{k}\right)=\frac{\Gamma(m+k).\Gamma(n+m)}{\Gamma(m+n+k).\Gamma(m)}.2F_{1}\left(m+k\,,n+m\,,n+m+k\,,1-\frac{\theta_{2}m}{\theta_{1}n}\right)$$

$$Var(r)=\frac{m(m+1)}{(n+m).(n+m+1)}.2F_{1}\left(m+2\,,n+m\,,n+m+2\,,1-\frac{\theta_{2}m}{\theta_{1}n}\right)$$

$$-$$

$$\left[\frac{m}{(n+m)}.\left(\frac{\theta_{2}m}{\theta_{1}n}\right)^{m}.2F_{1}\left(m+1\,,n+m\,,n+m+1\,,1-\frac{\theta_{2}m}{\theta_{1}n}\right)\right]^{2}$$

where

$$2F_{1}(a\,,b\,,c\,,\tau)=\sum_{j=0}^{\infty}\frac{(a)_{j}.(b)_{j}}{(c)_{j}}.\frac{\tau^{j}}{j!}\qquad;\qquad(a)_{j}=\frac{\Gamma(a+j)}{\Gamma(j)}$$

4 – Application

Here, we will use a real data set to get the results of the ER distribution, consider an uncensored data set corresponding an uncensored data set from Nichols and Padgett (2006) consisting of 100 observations on breaking stress of carbon fibers (in Gba):

3.7, 2.74, 2.73, 2.5, 3.6, 3.11, 3.27, 2.87, 1.47, 3.11,4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.9, 3.75, 2.43, 2.95, 2.97, 3.39, 2.96, 2.53,2.67, 2.93, 3.22, 3.39, 2.81, 4.2, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56, 3.56, 3.15, 2.35, 2.55, 2.59,2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19,1.57, 0.81, 5.56, 1.73, 1.59, 2, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69,1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.7, 2.03, 1.8, 1.57, 1.08, 2.03, 1.61, 2.12,1.89, 2.88, 2.82, 2.05, 3.65 .

These data are used here only for illustrative purposes. The required numerical evaluations are carried out using the Package of Mathcad software.

Tables (1) provide the MLEs (with corresponding standard errors in parentheses) of the model parameters. The model selection is carried out using the AIC (Akaike information criterion), the BIC (Bayesian information criterion) and the CAIC (consistent Akaike information criteria):

$$\mathrm{AIC} = -2\ell(\hat{\theta}) + 2q\ ,$$

$$\mathrm{BIC} = -2\ell(\hat{\theta}) + q\log(n), \qquad \mathrm{CAIC} = -2\ell(\hat{\theta}) + \frac{2qn}{n-q-1}$$

where $\ell(\hat{\theta})$ denotes the log-likelihood function evaluated at the maximum likelihood estimates, q is the number of parameters, and n is the sample size.

<u>Table</u>(1). MLEs of the model parameters and the statistics AIC, BIC and CAIC with True Value (5.2, 4.87 and 1.39) for (θ_1, θ_2 and σ^{2}) respectively

Model	Estimates			Statistic		
	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\sigma}^2$	AIC	BIC	CAIC
EX – R	5.2	4.87	1.459	536.052	536.052	536.302
Rayleigh	--	--	2.352	378.655	378.655	378.905

Since the values of the AIC, BIC and CAIC are greater for the EX-R distribution compared with those values of the other model, the EX-R distribution seems to be a very competitive model to these data.

5- Concluding Remarks

The well-known three-parameter exponentiated Rayleigh distribution is extended by introducing extra shape parameter. This is achieved by taking (2) as the baseline cumulative distribution of the generalized class of exponentiated Rayleigh distribution. This model includes as special sub-models Generalized Rayleigh distribution. The estimation of the model parameters is approached by maximum likelihood and the observed information matrix is obtained. An application to a real data set indicates that the fit of the new model is superior to the fits of its principal sub-models. We hope that the proposed model may be interesting for a wider range of statistical research.

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